Homework 5

MA 123 A2, Summer I 2010

Be first to report a math error for extra credit.

Read Stewart sections 3.1-3.7. Alternatively, read Basics of Differentiation at `http://en.wikibooks.org/wiki/Calculus/Differentiation`.
Both sources contain ample selections of practice exercises.

Exercise 1. Differentiate the function $f(x) = 2x^{-3/4}$.

$[-\frac{3}{2}x^{-7/4}]$

Exercise 2. Differentiate the function $y(x) = \dfrac{x^2+4x+3}{\sqrt{x}}$.

$[\frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2x\sqrt{x}}]$

Exercise 3. Find a second degree polynomial $p(x)$ such that $p(2) = 5$, $p'(2) = 3$, and $p''(2) = 2$.

$[x^2 - x + 3]$

Exercise 4. Find the points on the curve $y(x) = 2x^3 + 3x^2 - 12x + 1$ where the tangent line is horizontal.

$[(-2, 21),\ (1, -6)]$

Exercise 5. Differentiate the function $\frac{e^x}{x^2}$.

$[(x-2)e^x/x^3]$

Exercise 6. Differentiate the function $g(x) = \dfrac{3x-1}{2x+1}$.

$[5/(2x+1)^2]$

Exercise 7. Differentiate the function $\dfrac{x^3}{1-x^2}$.

$[\frac{x^2(3-x^2)}{(1-x^2)^2}]$

Exercise 8. Differentiate the function $\dfrac{A}{B+Ce^x}$.

$[-ACe^x/(B+Ce^x)^2]$

Exercise 9. Find $f'(x)$ and $f''(x)$ if $f(x)=x^4e^x$.

$[(x^4+4x^3)e^x,\ (x^4+8x^3+12x^2)e^x]$

Exercise 10. Plot the curve $y(x)=x/(x+1)$. Find all the tangent lines to the curve $y(x)$ which contain the point $(1,2)$, and find the points where these tangents touch the curve.

[The points of tangency are $(-2\pm\sqrt{3},\frac{1}{2}(1\mp\sqrt{3}))$]

Exercise 11. Differentiate the function $f(t)=c\cos(t)+t^2\sin(t)$.

$[-c\sin(t)+t(t\cos(t)+2\sin(t))]$

Exercise 12. Differentiate the function $y(x)=\dfrac{x}{2-\tan(x)}$.

$[\frac{2-\tan(x)+x\sec^2(x)}{(2-\tan(x))^2}]$

Exercise 13. Find $f^{(1025)}(x)$ if $f(x)=\sin(x)$.

Exercise 14. Find all points where the curve $y(x)=x+\cos(x)$ has a horizontal tangent line.

$[\frac{\pi}{2}+2\pi n$ for each $n\in\mathbb{Z}]$

Exercise 15. Differentiate the function $y(x)=\sqrt{1-2x}$.

$[-1/\sqrt{1-2x}]$

Exercise 16. Differentiate the function $f(x)=\cos(a^3+x^3)$.

$[-3x^2\sin(a^3+x^3)]$

Exercise 17. Differentiate the function $f(x)=xe^{-kx}$.

$[e^{-kx}(-kx+1)]$

Exercise 18. Differentiate the function $f(x)=2^{\sin(\pi x)}$.

$[2^{\sin(\pi x)}(\pi\ln 2)\cos(\pi x)]$

Exercise 19. Differentiate the function $f(x) = \cos\left(\sqrt{\sin(\tan(\pi x))}\right)$.

$\left[\frac{-\pi\cos(\tan(\pi x))\sec^2(\pi x)\sin\sqrt{\sin(\tan(\pi x))}}{2\sqrt{\sin(\tan(\pi x))}}\right]$

Exercise 20. Find the equation of the line tangent to the curve $y(x) = \sin(\sin(x))$ at the point $(\pi, 0)$.

$[y = -x + \pi]$

Exercise 21. Find $y^{(1717)}(x)$ if $y(x) = \cos(2x)$.

$[-2^{1717}\sin(2x)]$

Exercise 22. Find $y^{(1717)}(x)$ if $y(x) = xe^{-x}$.

Exercise 23. Differentiate the function $f(x) = \sqrt[5]{\ln x}$.

$\left[\frac{1}{5x\sqrt[5]{(\ln x)^4}}\right]$

Exercise 24. Differentiate the function $f(x) = \ln|2 - x - 5x^2|$.

$\left[\frac{10x+1}{5x^2+x-2}\right]$

Exercise 25. Find $\frac{dy}{dx}$ by implicit differentiation if $x^3 + y^3 = 1$.

$[y' = -x^2/y^2]$

Exercise 26. Find $\frac{dy}{dx}$ by implicit differentiation if $4\cos(x)\sin(y) = 1$.

$[y' = \tan(x)\tan(y)]$

Exercise 27. Find $\frac{dy}{dx}$ by implicit differentiation if $e^{x/y} = x - y$.

$\left[y' = \frac{y(y-e^{x/y})}{y^2 - xe^{x/y}}\right]$

Exercise 28. Use implicit differentiation to find an equation of the tangent line to the ellipse $x^2 + xy + y^2 = 3$ at the point $(1, 1)$.

$[y = -x + 2]$

Exercise 29. Find $f'(x)$ if $f(x) = x^x$, $x \geqslant 0$.