Homework 6

MA 123 A2, Summer I 2010

Be first to report a math error for extra credit.

Read Stewart sections 4.1-4.3, 4.5, 4.6. Alternatively, read about extrema, optimization, and related rates at
`http://en.wikibooks.org/wiki/Calculus/Differentiation`.
Both sources contain ample selections of practice exercises.

Exercise 1. A cylindrical tank with radius 5 m is being filled up with water at a rate of 3 m^3/min. How fast is the height of the water increasing?

[$\frac{3}{25\pi}$ m/min]

Exercise 2. A snowball melts so that the surface area decreases at a rate of 1 cm/min. Find the rate at which the diameter decreases when the diameter is 10 cm.

[$\frac{1}{20\pi}$ cm/min]

Exercise 3. A plane flying horizontally and straight at an altitude of 1 km and a speed of 500 km/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when the plane is 2 km away from it.

[$250\sqrt{3}$ km/h]

Exercise 4. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3 m away from the wall, it slides away from the wall at a rate of 0.2 ms. How long is the ladder?

[5 m]

Exercise 5. Find critical points of the function $f(x) = x^3 + 3x^2 - 24x$.

[$-4, 2$]

Exercise 6. Find critical points of the function $f(x) = x^2 e^{-3x}$.

[$0, 2/3$]

Exercise 7. Find critical points of the function $f(x) = \dfrac{x-1}{x^2-x+1}$.

$[0,2]$

Exercise 8. Find the absolute maximum and the absolute minimum values of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-2,3]$.

$[f(-1) = 8, f(2) = -19]$

Exercise 9. Find the absolute maximum and the absolute minimum values of $f(x) = x\sqrt{4-x^2}$ on the interval $[-1,2]$.

$[f(\sqrt{2}) = 2, f(-1) = -\sqrt{3}]$

Plotting the functions given in the exercises below is a great way to improve your understanding of the derivative. Beyond plotting the parameters you have to find, it helps also to plot a function's asymptotic behavior and its intercepts, if you can find them.

In the following exercises, find intervals on which f is increasing (a), decreasing (b); find local extreme values of f (c); find intervals on which f is concave up (d), down (e); find the inflection points (f). Plot the function.

Exercise 10. $f(x) = 2x^3 + 3x^2 - 36x$.

[(a) $(-\infty,-3),(2,\infty)$, (b) $(-3,2)$, (c) max $(-3,81)$, min $(2,-44)$, (d) $(-1/2,\infty)$, (e) $(-\infty,-1/2)$, (f) $(-1/2,37/2)$]

Exercise 11. $f(x) = e^{2x} + e^{-x}$.

[(a) $(-\ln 2/3,\infty)$, (b) $(-\infty,-\ln 2/3)$, (c) min $(-\ln 2/3, 2^{-2/3} + 2^{1/3})$, (d) $\mathbb{R}$]

Exercise 12. $f(x) = x\sqrt{x+3}$.

[(a) $-2,\infty$, (b) $(-3,-2)$, (c) min $(-2,-2)$, (d) $(-3,\infty)$]

Exercise 13. $f(x) = \sin(x) + \cos(x)$, $x \in [0,2\pi]$.

[(a) $(0,\pi/4),(5\pi/4,2\pi)$, (b) $(\pi/4,5\pi/4)$, (c) max $(\pi/4,\sqrt{2})$, min $(5\pi/4,-\sqrt{2})$, (d) $(3\pi/4,7\pi/4)$, (e) $(0,3\pi/4),(7\pi/4,2\pi)$, (f) $(3\pi/4,0),(7\pi/4,0)$]

In the following exercises, find the given limit.

Exercise 14. $\lim_{x\to 1} \frac{x^2-1}{x^2-x}$.

[2]

Exercise 15. $\lim_{t\to 0} \frac{e^t-1}{t^3}$.

[∞]

Exercise 16. $\lim_{t\to 0} \frac{e^{3t}-1}{t}$.

Exercise 17. $\lim_{t\to 0} \frac{5^t-3^t}{t}$.

[$\ln \frac{5}{3}$]

Exercise 18. $\lim_{x\to 0^+} x^{x^2}$.

[1]

Exercise 19. $\lim_{x\to \infty} x^{x^{-1}}$.

[1]

Exercise 20. If 1200 cm^2 of material is available to build a box with a square base and an open top, find the largest possible volume of the box.

[4000 cm^3]

Exercise 21. Show that out of all rectangles with a given area, the one with the smallest perimeter is the square.

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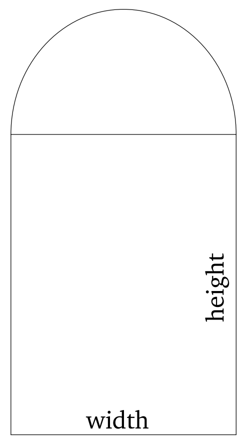
Exercise 22. Find the points on the ellipse $4x^2+y^2=4$ that are farthest away from the point $(1,0)$.

[$(-\frac{1}{3}, \pm\frac{4}{3}\sqrt{2})$]

Exercise 23. Find an equation of the line through the point $(3, 5)$ that cuts off the least area from the first quadrant.

Exercise 24. A Norman window is a rectangle surmounted by a semicircle, so that the diameter of the semicircle is equal to a side of the rectangle. If the perimeter of the window is 30 meters, find the dimensions of the rectangle that would maximize the area of the window (and so also the amount of the light admitted).



[Width $60/(4+\pi)$ and height $30/(4+\pi)$]

Exercise 25. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other one into an equilateral triangle. How should the wire be cut so that the total area enclosed is (a) maximum? (b) minimum?

[(a) 10 m for the square, (b) $40\sqrt{3}/(9+4\sqrt{3})$ m for the square]