Midterm

MA 123 A2, Summer I 2010

This test is closed-books and closed-notes. No calculators or cellphones are allowed. There are 19 problems, all together worth 100 points.

LAST NAME ____

FIRST NAME ____

Problem 1 (6 points). Find the domain and the range of the function

$$f(x) = \sqrt{16 - x^2}.$$

Problem 2 (6 points). Find the domain and the range of the function

$$f(x) = 3 + \cos(2x).$$

Problem 3 (4 points). Determine if the function $f(x) = e^{-x^2}$ is even, odd, or neither.

Problem 4 (4 points). Prove that the function $f(x) = 1 + \sin(x)$ is neither even nor odd.

Problem 5 (6 points)**.** Solve the given equation for x:

$$e^{x^2-1} - 17 = 0.$$

Problem 6 (4 points)**.** Solve the given equation for x:

$$\ln(x^2) = \ln(x) + 1.$$

Problem 7 (6 points)**.** Let $f(x) = \sin(x^2)$ and $g(x) = e^{x+1}$. Find $f \circ g$, $g \circ f$, and $g \circ g$.

Problem 8 (4 points)**.** Let

$$H(x) = \frac{\sin(x) + 1}{\sin(x) - 1}.$$

Find functions $f(x)$ and $g(x)$ such that $H(x) = (f \circ g)(x)$ for all $x \in \operatorname{dom} H$. Avoid trivial solutions where $f(x)$ or $g(x) = x$.

Problem 9 (4 points)**.** Use the Intermediate Value Theorem to show that the equation

$$e^{-x^2} = x$$

has a root in the interval $(0, 1)$.

Problem 10 (6 points)**.** Find all intervals on which the function

$$f(x) = \frac{e^{\sin(x)}}{x^2 - x - 2}$$

is continuous.

Problem 11 (6 points)**.** Find $\lim_{x\to 0^-} \dfrac{x^3+x^2}{2x^2+x^3}$ or show that the limit does not exist.

Problem 12 (6 points)**.** Find $\lim_{x\to 1} \left(x^3 + 5x - \dfrac{1}{2-x} \right)$ or show that the limit does not exist.

Problem 13 (6 points)**.** Find $\lim_{x\to 1} \dfrac{x^2-1}{x^2+2x-3}$ or show that the limit does not exist.

Problem 14 (6 points)**.** Find $\lim_{x\to-\infty} \frac{x^2+2x+1}{3x^2+1}$ or show that the limit does not exist.

Problem 15 (6 points)**.** Find $\lim_{x\to\infty} \frac{3x^2+4x}{\sqrt{x^4+2}}$ or show that the limit does not exist.

Problem 16 (6 points)**.** Let $f(x) = \frac{1}{x}$. Use the definition of the derivative to find $f'(2)$.

Problem 17 (6 points)**.** Let $f(x) = 2\sqrt{2x+1}$. Use the definition of the derivative to find $f'(x)$.

Problem 18 (8 points)**.** The upper half of the unit circle can be represented as the graph of the function $f(x) = \sqrt{1-x^2}$, $x \in [-1,1]$. Use the definition of the derivative to find the equation of the tangent line to the graph of f at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$. Plot $f(x)$ and the tangent line you've found.

Problem 19 (0 points)**.** The equation of a circle in the Cartesian plane is

$$x^2 + y^2 = r^2,$$

where r is the constant radius. Can you write an equation for a square? How about a diamond?