PROBABILITY LAWS HOMEWORK.

1 (6 points). Suppose that A and B are events with:

- $P(A) = 0.15$
- $P(A \cap B) = 0.03$
- $P(A|B) = 0.12$

(a) Show that $P(B) = 0.25$

(b) Assuming that $P(B) = 0.25$, find $P(A \cup B)$

(c) Find $P(B|A)$

(d) Are events A and B independent? Explain. **Yes / No**

(e) Are events A and B mutually exclusive? **Yes / No**

1. E and F are mutually exclusive events with $P(E) = 0.4$ and $P(F) = 0.5$.
Find $P(E \mid F)$.

2. J and K are independent events. $P(J \mid K) = 0.3$.
Find $P(J)$.

3. Q and R are independent events with $P(Q) = 0.4$ and $P(Q \cap R) = 0.1$.
Find $P(R)$.

In the following questions, let U and V are mutually exclusive events, with $P(U) = 0.26$ and $P(V) = 0.37$.

4. Find $P(U \mid V)$.

5. Find $P(U \cap V)$.

6. Find $P(U \cup V)$.

United Blood Services is a blood bank that serves more than 500 hospitals in 18 states. According to their website, a person with type O blood and a negative Rh factor (Rh-) can donate blood to any person with any blood type. Their data show that 43% of people have type O blood and 15% of people have Rh-factor; 52% of people have type O or Rh- factor.

7. Find the probability that a person has both type O blood and the Rh- factor.

8. Find the probability that a person does not have both type O blood and the Rh- factor.

Suppose that 48% of all California's registered voters prefer life in prison without parole over the death penalty for a person convicted of first degree murder. Among California's Latino registered voters, 55% prefer life in prison without parole over the death penalty for a person convicted of first degree murder. 37.6% of all Californians are Latino. For the following problems, let:

$J =$ Californian registered voters preferring life in prison without parole over the death penalty for a person convicted of first degree murder.

$L =$ Latino Californians

The experiment is to sample one Californian at random.

9. Find $P(J)$.

10. Find $P(L)$.

11. Find $P(J \mid L)$.

12. Find $P(L \cap J)$.

13. Are L and J independent events? Show why or why not.

14. Find $P(L \cup J)$.

15. Are L and J mutually exclusive events? Show why or why not.

16. Find $P(J \mid J)$.

At a college, 72% of courses have final exams and 46% of courses require research papers. Suppose that 32% of courses have a research paper and a final exam.

17. Find the probability that a course has a final exam or a research project.

18. Find the probability that a course has neither of these two requirements.

Suppose that you have eight cards. Five are green and three are yellow. The five green cards are numbered 1, 2, 3, 4, and 5. The three yellow cards are numbered 1, 2, and 3. The cards are well shuffled. You randomly draw one card.

$G =$ card drawn is green.

$E =$ card drawn is even-numbered.

19. List the sample space.

20. Find $P(G)$.

21. Find $P(G|E)$.

22. Find $P(G \cap E)$.

23. Find $P(G \cup E)$.

24. Are G and E mutually exclusive? Why or why not?

Toss two fair six-sided dice, one blue and the other one red. Each die will show a random integer between from 1 to 6.

Let $F =$ the sum of the two dice is 4.
Let $T =$ one or the other die shows 3.
Let $D =$ the dice show different numbers.
Let $E =$ both dice show even numbers.

25. How many outcomes are in the sample space for this experiment?

26. Find $P(F)$.

27. Find $P(T)$.

28. Find $P(D)$.

29. Find $P(E)$.

30. Are the events D and E independent? Why or why not?

31. Are the events D and E mutually exclusive? Why or why not?

ANSWERS.

1. 0

3. 0.25

5. 0

7. 0.06

9. 0.48

11. 55%

12. 0.2068

13. The events are not independent:
$P(L)P(J) = .376 \times .48 = 0.18048$, and
$P(L \cap J) = 0.2068 \neq P(L)P(J)$

14. 0.6492

15. $P(L \cap J) = 0.2068 \neq 0$, so the events are not mutually exclusive.

16. 1

17. 0.86

19. $\{g_1, g_2, g_3, g_4, g_5, y_1, y_2, y_3\}$

21. 2/3

23. 3/4

26. 1/12

28. 5/6

30. The events are not independent:
$P(D)P(E) = 5/6 \times 1/4 = 5/24$, and

$$P(D \cap E) = 1/6 \neq P(D)P(E)$$