DISCRETE RV HOMEWORK.

A company wants to evaluate its attrition rate, in other words, how long new hires stay with the company. Over the years, they have established the following probability distribution. Here X is a random variable that is equal to the number of years a new hire stays with the company before moving on, and $P(X = x)$ is the probability that the new hire stays for x years.

x	$P(X = x)$
0	0.12
1	0.18
2	0.30
3	0.15
4	
5	0.10
6	0.04

1. How likely is a new hire to stay with the company for 4 years?

2. How likely is a new hire to stay with the company for 5 or 6 years?

3. How likely is a new hire to stay with the company for an odd number of years?

4. Find $P(X \leq 3)$

5. Find $P(X > 2)$

6. Find EX, the expected number of years a new hire will stay with the company.

7. Construct the cumulative probability distribution $P(X \leq x)$

Isabel tosses a fair coin until she observes Tails, as many times as it takes. Let the random variable T count the total number of tosses. The probability mass function of T can be expressed by

$$P(T = t) = \frac{1}{2^t}$$

where t is a positive integer. So, for example,

$$P(T = 1) = 1/2^1 = 0.5$$
$$P(T = 10) = 1/2^{10} = 0.0009765625$$

This random variable has an infinite pmf, since any positive integer outcome is possible. However, the sum of the (infinitely many) pmf entries is still 1.

8. Fill out the **shown portion** of the table with pmf and cdf values.

t	$P(T = t)$	$P(T \leq t)$
1		
2		
3		
4		
5		
...	...	...

9. Find the probability that 3 or fewer tosses will be observed.

10. Find $P(2 \leq T \leq 5)$

11. Find $P(2 < T < 5)$

12. Find the probability that more than 5 tosses will be observed.

13. You are playing a game by drawing a card from a standard 52 card deck and replacing it. If the card is a face card, you win $30. If it is not a face card, you pay $2. There are 12 face cards in the standard deck. What is the expected value for the profit in this game?

14. You buy a lottery ticket to a lottery that costs $10 per ticket. There are only 100 tickets available to be sold in this lottery. In this lottery there are one $500 prize, two $100 prizes, and four $25 prizes. Find your expected gain or loss from buying one ticket.

Carmen and Diego are playing a game where they toss a pair of fair 4-sided dice, one blue and the other red, with sides labeled 1, 2, 3, and 4. If the dice show the same number, then Carmen wins $5 from Diego. If the dice show different numbers, then Carmen loses $1 to Diego.

15. Construct a sample space (that is, the set of outcomes) for this game.

16. Construct the probability mass function for Carmen's profit.

17. Find the average amount Carmen wins or loses every time she plays the game.

18. Who is the long-term winner in this game?

ANSWERS.

1. 0.11

2. 0.14

3. 0.43

4. 0.75

5. 0.4

6. 2.41

7.

x	$P(X \leq x)$
0	0.12
1	0.30
2	0.60
3	0.75
4	0.86
5	0.96
6	1.00

8.

t	$P(T = t)$	$P(T \leq t)$
1	0.5	0.5
2	0.25	0.75
3	0.125	0.875
4	0.0625	0.9375
5	0.03125	0.96875

9. 0.875

10. 0.46875

11. 0.1875

12. 0.03125

13. $ 5.384615

14. $ −2

15. There are 16 outcomes in total, and 4 of them show the same number on the dice:

1,1	1,2	1,3	1,4
2,1	**2,2**	2,3	2,4
3,1	3,2	**3,3**	3,4
4,1	4,2	4,3	**4,4**

16.

x	P(X=x)
−1	0.75
5	0.25

17. $ 0.50

18. Carmen