

CLT HOMEWORK

TEXT: 7.1, 7.2, 7.3

LAST NAME	FIRST NAME	DATE
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Suppose that the mean length of a 2018 music track is 171 seconds, with the standard deviation of 34 seconds. A researcher takes a random sample of 7 popular music tracks produced in 2018 and measures the sample mean $\bar{Y}$.

1. Assuming that the Central Limit Theorem can be applied to this population and this sample size, what is the approximate distribution of $\bar{Y}$, the sample mean?
2. What are the chances that the sample mean is at most 140 seconds?
3. What is the 10th percentile of the sample mean?
4. What are the chances that the sample mean is between 150 and 160 seconds?

The mean body weight of an adult American is estimated to be 80.7 kg (Walpole 2005), with standard deviation 12 kg (Floud 1998). Take a simple random sample of 20 adult Americans and measure the mean weight in the sample $\bar{W}$.

5. Assuming that the Central Limit Theorem can be applied to this population and this sample size, what is the approximate distribution of $\bar{W}$?
6. How likely is the mean weight in the sample to be greater than 90 kg?
7. Find $P(80 < \bar{W} < 85)$
8. Suppose that a certain elevator can safely transport weights up to 1500 kg. What are the chances that a simple random sample of 20 people will overload the elevator?

A random sample of 120 lady bugs is chosen out of the population where 38% of individuals have dark spots, while the rest are plain red.

9. What is the distribution of the random variable X , which counts the number of spotted lady bugs in the sample?

10. What is the probability that at least 42 lady bugs in the sample are spotted?

11. Find the mean of X

12. Find the standard deviation of X

13. Describe Y , the normal approximation of X .

14. Use the normal approximation (with continuity correction) to compute the probability of the event where at least 42 lady bugs in the sample are spotted.

According to [census.gov](https://www.census.gov), the percentage of U.S. families with their own children under 18 in the household was 40% in 2022. Suppose we draw a simple random sample of 350 U.S. families and let C be the number of families in the sample with children under 18 in the household.

15. Is the random variable C discrete or continuous?
16. Describe the distribution of the random variable C .
17. Find $P(C \leq 120)$.
18. Find μ_C , the expected number of families with children in the sample.
19. Find σ_C , the standard deviation for the number of children in the sample.
20. Use a normal approximation with continuity correction to estimate $P(C \leq 120)$.

Studies suggest that about 88% of the world human population is right-handed. Take a random sample of size $n = 180$ and let X be the number of right-handed individuals in the sample.

21. What is the distribution of X ?

22. Find the expected number of right-handed individuals in the sample.

23. Find the standard deviation for the number of right-handed individuals in the sample.

24. Find the probability that at most 150 individuals in the sample are right-handed.

25. Find the sample proportion $\hat{p}$ for a sample of size 180, which has 150 right-handed individuals.

26. What is the approximate distribution of the sample proportion $\hat{P}$?

27. Use the CLT to approximate the likelihood of the sample proportion being at most $\hat{p}$, and compare the result with the exact binomial probability in part (d).

ANSWERS

1. $\bar{Y} \sim N(\mu = 171, \sigma = 12.85079)$
3. 154.531 seconds
5. $\bar{W} \sim N(\mu = 80.7, \sigma = 2.683282)$
7. 0.5483845
9. $X \sim \text{Binom}(n = 120, p = 0.38)$
11. 45.6
13. $Y \sim N(\mu = 45.6, \sigma = 5.317142)$
15. Discrete
17. 0.01607099
19. 9.165151
21. $X \sim \text{Binom}(n = 180, p = 0.88)$
23. 4.359817
25. 0.8333333
27. 0.02700934