

## HYPOTHESIS TESTING PROJECT

This document provides the guidelines for a hypothesis testing paper, as well as examples. While you are strongly encouraged to imitate the given examples in form, be advised that your choice of population, variable, and/or the sampling procedure should be original enough if you want to get full credit for this assignment.

### STATISTICAL STUDY PROPOSAL FORMAT

**Goal.** State what population parameter you want to study. State the null and the alternative hypotheses, and stipulate the desired confidence level  $\alpha$ .

**Population.** Describe the intended population precisely.

**Sampling frame.** Describe the sampling frame you will be using for taking your sample. Describe in detail how exactly you will select individuals for measurement: will you be picking random items from a large list? Or measuring convenient individuals at randomly selected times and locations? Recall our discussion of multistage sampling, and identify which, if any of these sampling techniques you will be using.

**Sampling method.** Imagine yourself performing the data collection, and describe in minute detail what you will do and how. State the intended/expected sample size. The main idea behind this section is to provide a description of a procedure so detailed, that anyone can repeat your experiment just the way you conducted it. Another reason to be thorough here is to lay bare all the limitations of your chosen sampling technique.

**Discussion.** Provide constructive criticism of your proposal. In other words, enumerate the flaws which are present by design. There is no shame in finding flaws: almost every time you can blame them on either the lack of resources or an ethical dilemma or whatnot. There is, however, a great amount of shame in being willfully blind to the consequences of your own choices, and biases inherent in your sampling procedure. Think of it this way: if you don't point out the built-in flaws now, someone will surely bring them up later in an attempt to discredit your study.

## STATISTICAL STUDY PUBLICATION FORMAT

**Data analysis.** Provide full raw data as a separate spreadsheet file, using a portable file format. State the relevant sample statistics. State the 95% confidence interval for the parameter you studied and the margin of error. Most importantly, state the details and the results of the testing procedure: which distribution was assumed for the test statistic, the critical values, the test statistic itself, the p-value, and the conclusion.

**Discussion.** This is your time to be both creative and reflective. Some of the points you may want to bring up:

- Mention all the things that went wrong, but also pat yourself on the back for all the things that went right.
- If you encountered unexpected difficulties and had to deviate from your proposal in any way, describe all of that in detail, and explain how that might have affected the data and the conclusion.
- Re-examine the assumptions and discuss how well they were met in your sample. For proportion studies, you would want a large sample and a much larger population, especially if the point estimate is extreme. For mean and standard deviation studies, you would like to see an approximately normal population or else a very large sample size.

#### EXAMPLE: TEST FOR THE PROPORTION PROPOSAL

**Goal.** The purpose of this study is to test the hypothesis that less than half of Sacramento CA residents own a pickup truck, at 10% significance level, by obtaining an estimate of  $p$ , the proportion of Sacramento CA residents who own a pickup truck.

$$H_0 : p = 0.5$$

$$H_1 : p \neq 0.5 \text{ (two-tail test)}$$

**Population.** Every person 18 years of age or older, residing within the Sacramento metropolitan area, will be considered for this study.

**Sampling frame.** Individuals will be chosen by taking a random sample of phone numbers with area codes 916 and 279.

**Sampling method.** We will take a stratified sample, with one stratum per area code. In each stratum, 40 phone numbers will be sampled. R software will be used to generate random numbers. A list of 40 random, uniformly distributed phone numbers will be generated with

```
sample(1000000:9999999, 40)
```

(The range starts with 1000000 because phone numbers cannot begin with zero.)

Each selected phone number will be called during a weekend, with the following script:

- (a) Hi! How are you? My name is Ivan, I am a CRC college student taking an online Stat class, and I am conducting a non-profit statistical study of truck ownership patterns in Sacramento area. Would you like to participate by answering just three easy questions? My questions are non-personal, and the whole thing will take less than a minute of your time.
- (b) Are you at least 18 years old?
- (c) Do you currently reside within Sacramento metropolitan area?
- (d) Do you own at least one pickup truck?

Responses will be recorded as follows:

- If the phone number is invalid, or goes to voice mail, or a human respondent cannot be reached for any other reason, the response is NA.
- If the respondent answers negatively to either question (b) or question (c) or both, the response is NA.
- The response to question (d) is recorded as “yes”, “no”, or NA in the event if an unforeseen discrepancy occurs.

The final, adjusted sample size will be the number of all non-NA responses for both strata combined.

#### Discussion.

- (a) The sample is biased to have a preference for individuals who own phones and tend to answer cold calls.
- (b) Bias is introduced due to the possibility of respondents lying.
- (c) Bias is introduced because we are calling people during the weekend at convenient times, rather than at randomly distributed times.
- (d) Since we do not know how many responses will be NA, no particular sample size can be guaranteed.

EXAMPLE: TEST FOR THE PROPORTION PUBLICATION

**Data analysis.** Raw data is attached in a separate file: **sample-data-p-estimate.csv**

Adjusted sample size:  $n = 51$

Responses: 21 “yes”, 30 “no”.

Sample proportion (point estimate):  $\hat{p} = 0.4117647$

95% CI for population proportion using  $Z$  as the sampling distribution:

$$(0.2766934, 0.5468360)$$

Margin of error: 0.1350713

Using  $\alpha = 0.1$  and  $Z$  as the approximate distribution of the test statistic:

Critical value:  $-1.644854$

Test statistic:  $-1.260252$

P-value: 0.1037892

**Conclusion:** There is not enough evidence to reject  $H_0$  at 10% significance level. In other words, we cannot conclude that the proportion of Sacramento CA residents who own a pickup truck is lower than half.

**Discussion.** The sample size is small, but the sample proportion  $\hat{p}$  is close to 0.5, and

$$n\hat{p}(1 - \hat{p}) = 12.35$$

indicates that the sample size is large enough to justify the application of normal distribution.

## EXAMPLE: TEST FOR THE MEAN PROPOSAL

**Goal.** The purpose of this study is to test the hypothesis that the mean **girth of a tree** in Alhambra Triangle neighborhood of Sacramento CA is different from 500 mm, at 10% significance level, by obtaining an estimate of  $\mu$ , the mean girth.

$$H_0 : \mu = 500$$

$$H_1 : \mu \neq 500 \text{ (two-tail test)}$$

**Population.** Every tree within the shown area will be considered for the study, given that

- the height of a tree is 6 feet or more
- the tree grows on a patch of land open to general public

**Sampling frame.** Instead of selecting individuals, we will be selecting geographical locations (clusters), and then measuring nearby individuals (a sample of convenience). Each geographical location will be represented by a point on the map ( $x$  and  $y$  coordinates) where we will look for individuals to measure.

**Sampling method: choosing location.** We will take a cluster sample by selecting 5 random locations on the map of the Alhambra Triangle, walking over to these spots, and measuring a number of trees at each location. R will be used to generate random numbers.

To pick a location, we will print the map of the Alhambra Triangle, measure its dimensions, and then roll two random numbers: an  $x$  coordinate and a  $y$  coordinate. We will then mark this spot on the map with a circle.

For example, we will use a 12 by 6 cm map image. We need to roll 2 random numbers which are uniformly distributed in their respective ranges:

```
runif(1, 0, 12) # roll 1 number between 0 and 12
```

```
[1] 3.454436
```

```
runif(1, 0, 6) # roll 1 number between 0 and 6
```

```
[1] 2.633399
```



Map generated by Google

If the random location is within the Alhambra Triangle area, then we mark the location on the map, as seen above. If the random location is outside of the highlighted area, or overlaps with a formerly chosen location, then we discard both coordinates and roll again.

**Sampling method: individual measurement.** Once we have all 5 random locations marked on the map, we will travel to each one and attempt to make 8 individual measurements at each location. If the location is on an intersection, then we will take two measurements from each of the four corners. If the location is in the middle of a street, we will take 4 measurements from each side of the street. In all other circumstances we will measure 8 individual trees which are closest to the chosen coordinates.

To measure a single tree, we will use a metric measuring tape. The girth will be measured at 3 feet above the ground, and recorded as a single number measured in millimeters. If the tree is too thick to be measured with the tape, NA will be entered.

**Discussion.**

- (a) Chosen locations are large and imprecise, so picking the 8 closest trees may be difficult without resorting to a sample of convenience, resulting in bias.
- (b) Some locations, like the areas under the freeway, may be inaccessible or lacking in trees, resulting in a reduced sample size.
- (c) Bias will also be introduced by discarding trees with height below 6 feet, because they are too hard to distinguish from bushes. This will probably result in an over-estimate of the mean girth.

EXAMPLE: TEST FOR THE MEAN PUBLICATION

**Data analysis.** Raw data is attached in a separate file: **sample-data-mu-estimate.csv**

Adjusted sample size:  $n = 34$

Sample mean (point estimate):  $\bar{x} = 613.0588$  mm

Sample standard deviation:  $s = 296.2632$  mm

95% CI for population mean using  $t_{33}$  as the sampling distribution, in mm:

(509.6877, 716.4299)

Margin of error: 103.3711 mm

Using  $\alpha = 0.1$  and  $t_{33}$  as the approximate distribution of the test statistic:

Critical values:  $-1.69236, 1.69236$

Test statistic: 2.225186

P-value: 0.03301779

**Conclusion:** There is enough evidence to reject  $H_0$  at 10% significance level. In other words, we are confident that the mean tree girth in Alhambra Triangle is different from 500 mm.

**Discussion.** The sample size of 34 is rather small, but the sample appears to be approximately normal: the histogram is somewhat mound-shaped and symmetric, and the Q-Q plot shows that the bulk of the sample data follows normal quantiles.

